III. Principles and Background

Force measurement is an important part of many measurement systems. Pressure, drag, torque, and other parameters may be indirectly measured by measuring the forces applied to a device. One of the best ways to measure force is also an indirect method: measurement of the **strain** caused when an applied force induces stress in an object and thus deformation. This experiment relates the stress to the strain in a cantilever beam loaded with a point load.

The definition of stress is Force/Area. For a beam in tension, stress is simply related to axial force by

$$\sigma_{axial} = \frac{F_{axial}}{A_{axial\,cross\text{-}section}}$$

In many materials, stress relates linearly to strain over an elastic range with Hooke's law. For a simple **1-dimensional** stress state (non-zero stress only acting in **one** direction), it is given by the equation

$$\varepsilon_{axial} = \frac{\sigma_{axial}}{E}$$

where E is the material modulus of elasticity (or Young's modulus). This linear relationship ends at high stresses when the yield point is reached and plastic deformation occurs. For materials, where Hooke's law applies, we can measure strain to find stress through a linear equation.

Strain is a physical second-order tensor, (i.e. it has nine components, because of the symmetry of the tensor six of these components are unique: 3 normal strains and 3 shear strains), which describes the distortion of an elastic medium at a given point. On a free surface, the strain at a point may be described with three components (2 normal strains and 1 shear strain). These three components may be two perpendicular normal strains and the shear strain between these two directions. If the directions of principal strain are known, these directions and the two values of principal normal strain are sufficient to define the state of strain at that surface point. If these directions are not known, a strain rosette must be used to measure the state of strain at that point.

Extensional strain is defined as the change in length divided by the original length.

$$\varepsilon = \frac{\Delta L}{L}$$

Strain such as stress is a tensor with three normal strains and three shear strains defining the local deformation. Strain varies throughout an elastic object. When measuring strain, the strain is an average value over the original length L. Therefore the smaller the original length of the specimen, the more localized the measurement of the strain (Note: L here is any arbitrary length over which strain is measured, not to be confused with L, the beam length).

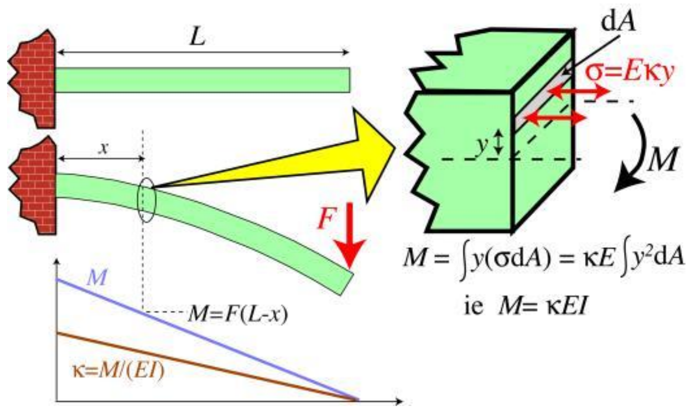
Transverse strain is defined as the strain transverse (normal) to the direction of reference. If the direction of reference is the axial beam direction, the transverse strain is normal to the axial direction. When a beam is subjected to positive axial strain, the beam cross-section narrows, thus giving rise to a negative transverse strain. For a one-dimensional stress state (stress only occurs in one direction, it is zero in the other directions) the relation of axial to transverse strain is defined as the Poisson's ratio. It is a material property and defines the amount of "contraction" of the specimen in the transverse direction under uniaxial stress. It is defined by the equation

$$\nu_p = -\frac{\varepsilon_{transverse}}{\varepsilon_{axial}}$$

Because strains are changes in dimension, normal surface strains are easily measured quantities. Many methods and devices are available to measure localized displacements. Other quantities such as stress, force, torque and pressure are often determined by measuring strain in a controlled device (transducer), and relating the strain to the quantity of interest.

Strain in Cantilever Beams

In this lab we'll be measuring strain on the surface of a cantilever beam. The applied load at the beam tip creates a bending moment M, which creates a normal stress and strain distribution in the beam cross section. The surface strain can be measured at the top or bottom surface of the beam by measuring the local change in length.



For the bending deformation of the cantilever beam shown, each cross sectional plane will undergo a rotation κ, which will produce strains and stresses in the cross section, growing linearly outward from the neutral axis of bending ($\varepsilon_x = \kappa y$). It can be shown that the axial stress, σ_{axial}, at the upper and lower surface (y = h/2) is given by:

$$\sigma_{axial} = \frac{M}{I}\frac{h}{2}$$

where *M* is the bending moment, *I* the moment of inertia and *h* the thickness of the beam. We calculate the second moment of area, *I* for a rectangular beam cross section from

$$I = \frac{bh^3}{12}$$

The maximum bending moment, *M*, at any position, *d = L-x* , relative to the location of the force, *F* is

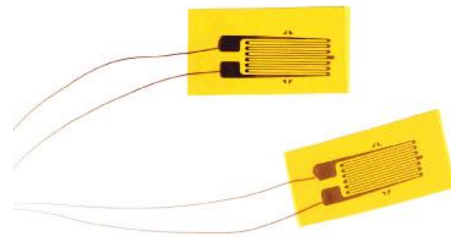
$$M = F * d$$

The axial strain, ε_{axial}, at the upper and lower surface is related to the stress through the material behavior, which is assumed to follow Hooke's law.

$$\varepsilon_{axial} = \frac{\sigma_{axial}}{E}$$

With these equations stress and strain can be determined from beam dimensions, material and load.

Metal Wire/Foil Strain Gage

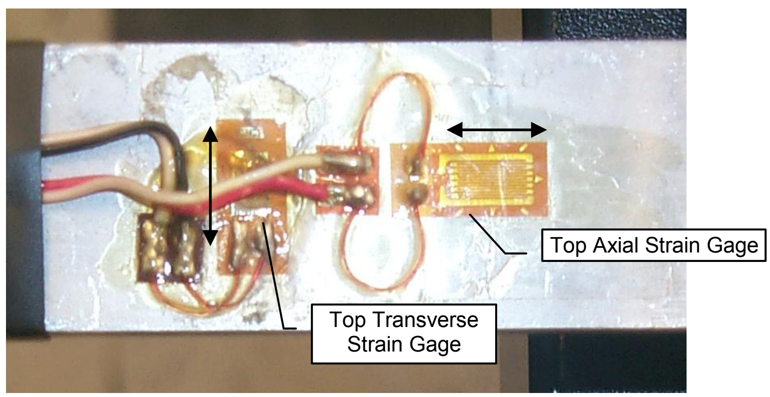


Wire is a convenient way to measure strain. When strained, a wire will increase in length and thin in cross-section. This causes an increase in resistance of the wire given a constant material resistivity ρ_e.

$$R_{wire} = \frac{\rho_e L}{A_{x\text{-}s}}$$

This effect is the basis for strain gages. Resistance-type strain gages may be constructed of either thin metal wire or a thin metal foil. Metal foil gages are the most common. They can be produced inexpensively in a variety of patterns by an etching process similar to that used in electronics manufacturing. The gages are available in lengths ranging from 0.008" to 4" and can be as inexpensive as $1 apiece. Generally, larger gages can have higher gage resistance and can operate at higher working voltages, giving better sensitivity. They also average strain over a large area, which may be a disadvantage, if localized strain measurements are needed. In contrast, smaller gages average out the strain over shorter lengths, thus providing a more precise value of the local strain. The nominal gage resistances are standardized to 120.5Ω, 350.5Ω, and 1000.5Ω.

The etched foil grid is very flexible and very thin. These properties minimize its influence on the object to which it is bonded. The metal foil is cemented to a thin, flexible sheet of plastic that acts as a base for the gage and provides electrical insulation between the metal gage and the object on which the gage is bonded. The strain is transferred from the specimen to the gage through the adhesive; therefore, extreme care must be taken in proper mounting. Lead wires are soldered to two tabs on the gage and extended to a remote location where the resistance measurement circuitry is located. A simple single element gage pattern is shown in Figure 1.
The location of the top strain gages is shown in the image to the right. The two strain gages are oriented to measure axial and transverse strain respectively



Gage Factor

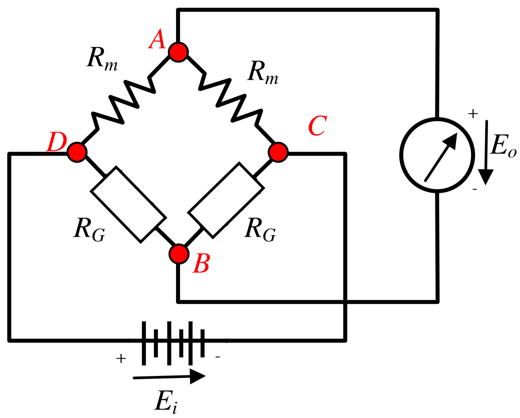
The gage factor, G_F, relates a unit change in length (i.e., strain) to a unit change in gage resistance. This can be expressed as:

$$\frac{\Delta R}{R} = G_F \frac{\Delta L}{L} = G_F \, \varepsilon$$

where R is the nominal resistance of the gage, 120.5 Ω for the gages in this lab, and ΔR is the change in resistance due to the strain.

The gage factor is a function of the gage material only, and it is not affected by the gage configuration and orientation. The gage factor is supplied by the manufacturer, and typically has a value of around 2.0 for most metal strain gages. The gages used in this lab have a nominal gage factor of $G_F = 2.09 \pm 5\%$.

A resistance-type strain gage measures strain by converting a change in length of the thin foil conductive path to a slight change in electrical resistance. The change in resistance is proportional to the change in length of the gage as shown above. Signal conditioning is needed to convert the very small change in gage resistance into a measurable change in voltage.



Wheatstone Bridge Circuit

One circuit that converts resistance to voltage is the Wheatstone bridge. Wheatstone bridge circuits are based on the principle of comparing the voltage drops across parallel resistance "legs". From Kirchhoff's laws the output voltage to input voltage ratio is found to be

$$\frac{E_0}{E_i} = \frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4}$$

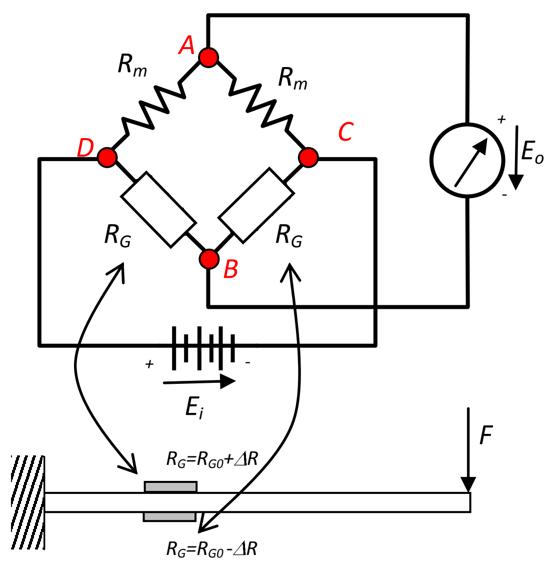
If the resistances in the bridge satisfy the "bridge balance" equation

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

then the bridge output voltage E_0 is zero. If even a small imbalance exists between the resistance ratios, the bridge generates a voltage output.

Wheatstone Half-Bridge Circuit

By replacing some of the resistors in the bridge by strain gages, a strain measurement circuit is obtained. There are several bridge circuits that distinguish themselves by the number of sensing resistors used. In the lab **a half-bridge** circuit is used as shown on the left. It uses two strain gauges on opposing arms of the bridge. By placing the strain gauges at the top and bottom of the beam, the strain gauge measure the tension and compression strain on the beam on the opposing arms.



Schematic of the Wheatstone Bridge circuit with two strain gauges with resistance R_g for the <u>half bridge</u> configurations.

Applying the general bridge equation to the half-bridge circuit yields an equation that relates the output voltage to the applied strain and gage factor (for a detailed derivation, see the lecture notes). Note that the actual values of the resistors drop out of the equation.

$$\frac{E_0}{E_i} = \frac{G_F}{2}\varepsilon$$